THREE-DIMENSIONAL HYPERSONIC GAS FLOW PAST SLENDER BODIES AT HIGH ANGLES OF ATTACK

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We consider three-dimensional hypersonic flow past bodies whose transverse dimensions are substantially smaller than their length. Making use of the smallness of a parameter characterizing the relative thickness of the body, it is possible to put the problem of flow past such a body approximately in a form that generalizes the similitude of hypersonic small-disturbance flow to the case of arbitrary angle of attack.

Approximate formulas are obtained for the calculation of the aerodynamic characteristics of slender bodies at high angles of attack, containing as unknowns only certain constants depending on the cross-sectional form of the body.

One approximate method of calculating hypersonic flows consists, as is well known, in considering flows past slender bodies whose surface is everywhere inclined at a small angle to the undisturbed stream. The velocity field about a slender body may be regarded as a small-disturbance field close to its surface. Here the angle of attack of the body must obviously also be small. Although the differential equations of hypersonic small-disturbance theory remain nonlinear, so that they cannot be solved in general form, it is possible within the framework of this theory to find certain general properties of hypersonic flows. The most useful ones are the analogy with unsteady gas motion (the equivalence principle, or "law of plane sections") [1,2,3] and the similarity rule for flow past affinely related bodies [4], an exposition of which may be found in the book [5].

With increasing angle of attack the perturbations produced in the stream even by a very slender body cease to be small, and the small-disturbance theory becomes inapplicable. However, as is shown below, in this case again the assumption of small relative thickness of the body permits a number of general conclusions to be drawn regarding the properties of three-dimensional hypersonic flows past such bodies at high angle of attack. These results may be regarded as a generalization of the equivalence principle and the similarity rule of small-disturbance theory to the case of arbitrary angle of attack. Here it is necessary to impose still one further limitation on the shape of the body: one considers bodies all of whose transverse dimensions are much smaller than their length. For example, in order to be able to apply the results of the present work to the calculation of the aerodynamic characteristics of wings at hypersonic speeds we must assume that, in addition to small thickness, they have extremely small span.

In considering hypersonic flows with finite perturbations of the velocity field and very intense shock waves it becomes important to consider real gas effects. The generalization of the present results to the case of flows of a real gas in thermodynamic equilibrium is given at the end of the paper.

1. Statement of the problem. We consider flow past a slender or elongated body placed in a uniform supersonic stream at angle of attack a. Let the greatest transverse dimension of the body be d, and its length be l. As a preliminary assumption we suppose that

$$\delta = \frac{d}{l} \ll 1 \qquad (1.1)$$

We will assume that the Mach number M_∞ of the undisturbed stream is significantly greater than unity, so that the following condition is satisfied:

$$M_\infty \delta \gtrsim 1 \qquad (1.2)$$

If the angle of attack is small ($a \lesssim \delta$) then the entire flow field between the shock wave and the surface of the body will comprise a region whose transverse dimension is of the order of the transverse dimension of the body (Fig. 1a). At high angles of attack ($a \gg \delta$) the disturbance field will, generally speaking, extend a finite distance from the body surface (Fig. 1b). However, it is easy to see that this refers only to parts that are behind the body (on the leeward side). The pressure field in this region is weak, and its influence on the remaining parts of the flow disappears by virtue of the hypersonic character of the cross-flow ($M_\infty \sin a \gg 1$). At the same time, the compressed part of the stream (whose pressure greatly exceeds the static pressure in the undisturbed stream) lies near the surface of the body, and its transverse dimensions are, as before, of the order of the transverse dimensions of the body.

Thus, even in the case of high angle of attack the problem of flow past a slender body reduces approximately to investigation of the flow near its surface. This circumstance leads to the possibility of its approximate analytical investigation.

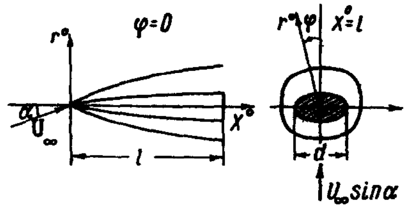

FIG. 1a.

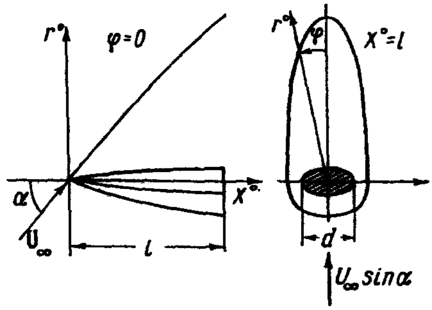

FIG. 1b.

2. **Equations and boundary conditions.** We introduce a system of coordinates x°, r°, ϕ° with the x° axis along the body, so that the angle of inclination of the surface with this axis is small. We will assume that the velocity vector $\mathbf{U}_\infty$ of the undisturbed stream lies in the plane $\phi = 0, \pi$ (Fig. 1).

We denote the components of the velocity vector $\mathbf{V}$ in this system of coordinates by u°, v°, w° respectively, and the pressure and density by p° and ρ°. We introduce dimensionless independent variables

$$x = \frac{x^\circ}{l}, \qquad r = \frac{r^\circ}{d}, \qquad \varphi = \varphi^\circ \tag{2.1}$$

and dimensionless dependent variables

$$u = \frac{u^\circ}{U_\infty \cos\alpha}, \qquad v = \frac{v^\circ}{U_\infty \sin\alpha}, \qquad w = \frac{w^\circ}{U_\infty \sin\alpha}$$

$$p = \frac{p^\circ}{\rho_\infty U_\infty{}^2 \sin^2\alpha}, \qquad \rho = \frac{\rho^\circ}{\rho_\infty} \tag{2.2}$$

The index ∞ refers throughout to conditions in the undisturbed stream.

In these variables the system of partial differential equations of gas dynamics takes the form

$$\begin{aligned}
\delta \cot\alpha\, u \frac{\partial(\cot\alpha u)}{\partial x} + v\frac{\partial(\cot\alpha u)}{\partial r} + \frac{w}{r}\frac{\partial(\cot\alpha u)}{\partial\varphi} &= -\delta\frac{1}{\rho}\frac{\partial p}{\partial x}\\
\delta \cot\alpha u \frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} + \frac{w}{r}\frac{\partial v}{\partial\varphi} - \frac{w^2}{r} &= -\frac{1}{\rho}\frac{\partial p}{\partial r}\\
\delta \cot\alpha\, u \frac{\partial w}{\partial x} + v\frac{\partial w}{\partial r} + \frac{w}{r}\frac{\partial w}{\partial\varphi} + \frac{vw}{r} &= -\frac{1}{\rho r}\frac{\partial p}{\partial\varphi}
\end{aligned} \tag{2.3}$$

$$\delta\cot\alpha u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial r}+\frac{w}{r}\frac{\partial\rho}{\partial\varphi}+\rho\left[\delta\frac{\partial(\cot\alpha u)}{\partial x}+\frac{\partial v}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial\varphi}+\frac{v}{r}\right]=0$$

$$\delta\cot\alpha u\frac{\partial S^\circ}{\partial x}+v\frac{\partial S^\circ}{\partial r}+\frac{w}{r}\frac{\partial S^\circ}{\partial\varphi}=0$$

where S° is the specific entropy of the gas, which we will regard as a function of pressure and density. For a perfect gas with constant specific heats

$$S^\circ=S_\infty\ln\left(\frac{1}{\gamma}M_\infty{}^2\sin^2\alpha\frac{p}{\rho^\gamma}\right)\tag{2.4}$$

where S_∞ is the specific entropy in the undisturbed stream, and γ is the ratio of specific heats of the gas. We now consider the boundary conditions for the problem.

Let $r = r_1(x, \phi)$ be the equation of the surface of the body, and $r = R(x, \phi)$ the equation of the shock wave surface. We denote by $\mathbf{n}_1$ and $\mathbf{n}_2$ unit vectors normal to these surfaces. In conformity with (2.1) we have

$$\mathbf{n}_1=\mu_1\left\{-\delta\frac{\partial r_1}{\partial x},1,-\frac{1}{r_1}\frac{\partial r_1}{\partial\varphi}\right\}\tag{2.5}$$

The condition of tangent flow at the body surface has the form

$$\mathbf{V}\mathbf{n}_1=0,\qquad\text{or}\quad v-w\frac{1}{r_1}\frac{\partial r_1}{\partial\varphi}=\delta\cot\alpha u\frac{\partial r_1}{\partial x}\tag{2.6}$$

For the unit vector $\mathbf{n}_2$ normal to the shock wave surface we have

$$\mathbf{n}_2=\mu_2\left\{-\delta\frac{\partial R}{\partial x},1,-\frac{1}{R}\frac{\partial R}{\partial\varphi}\right\},\qquad\mu_2=\left[1+\frac{1}{R^2}\left(\frac{\partial R}{\partial\varphi}\right)^2+\delta^2\left(\frac{\partial R}{\partial x}\right)^2\right]^{-\frac{1}{2}}\tag{2.7}$$

Two mutually orthogonal tangent vectors on this surface may be determined as

$$\mathbf{t}_{21}=\left\{0,\frac{1}{R}\frac{\partial R}{\partial\varphi},1\right\},\qquad\mathbf{t}_{22}=\left\{-1-\frac{1}{R^2}\left(\frac{\partial R}{\partial\varphi}\right)^2,-\delta\frac{\partial R}{\partial x},\frac{\delta}{R}\frac{\partial R}{\partial x}\frac{\partial R}{\partial\varphi}\right\}\tag{2.8}$$

The equations of conservation of mass, momentum, and energy relating quantities on the two sides of the shock surface have the form

$$\rho^\circ\mathbf{V}\cdot\mathbf{n}_2=\rho_\infty\mathbf{U}_\infty\cdot\mathbf{n}_2,\qquad\mathbf{V}\cdot\mathbf{t}_{21}=\mathbf{U}_\infty\cdot\mathbf{t}_{21},\qquad\mathbf{V}\cdot\mathbf{t}_{22}=\mathbf{U}_\infty\cdot\mathbf{t}_{22}\tag{2.9}$$

$$p^\circ+\rho^\circ(\mathbf{V}\cdot\mathbf{n}_2)^2=p_\infty+\rho_\infty(\mathbf{U}_\infty\cdot\mathbf{n}_2)^2,\qquad\frac{1}{2}(\mathbf{V}\cdot\mathbf{n}_2)^2+h^\circ=\frac{1}{2}(\mathbf{U}_\infty\cdot\mathbf{n}_2)^2+h_\infty$$

Here h° is the specific enthalpy of the gas, regarded as a function of pressure and density. For a perfect gas with constant specific heats

$$h^\circ=h_\infty\gamma M_\infty{}^2\sin^2\alpha\frac{p}{\rho}\tag{2.10}$$

where h_∞ is the specific enthalpy of the undisturbed stream.

Substituting into (2.9) the relation (2.10), the dimensionless dependent variables (2.2), the expression for the velocity vector in the undisturbed stream

$$\mathbf{U}_\infty = U_\infty (\cos\alpha,\ \sin\alpha\cos\varphi,\ -\sin\alpha\sin\varphi) \tag{2.11}$$

and the expressions for the vectors (2.7) and (2.8) we can, after some manipulation, write the system of boundary conditions on the shock wave surface in the form

$$\cot\alpha\, u = \cot\alpha + \delta(\cos\varphi - v)\frac{\partial R}{\partial x}, \quad (v - \cos\varphi)\frac{1}{R}\frac{\partial R}{\partial \varphi} + w + \sin\varphi = 0 \tag{2.12}$$

$$-\delta\cot\alpha\, u\frac{\partial R}{\partial x} + v - w\frac{1}{R}\frac{\partial R}{\partial\varphi} = \frac{\gamma-1}{\gamma+1}\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\frac{1}{R}\frac{\partial R}{\partial\varphi}\right) + \frac{2}{\gamma+1}\frac{1}{M_\infty^2\sin^2\alpha}\,\frac{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial\varphi}\right)^2 + \delta^2\left(\frac{\partial R}{\partial x}\right)^2}{-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\frac{1}{R}\frac{\partial R}{\partial\varphi}} \tag{2.13}$$

$$p = \frac{2}{\gamma+1}\,\frac{\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\frac{1}{R}\frac{\partial R}{\partial\varphi}\right)^2}{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial\varphi}\right)^2 + \delta^2\left(\frac{\partial R}{\partial x}\right)^2} - \frac{1}{\gamma}\,\frac{\gamma-1}{\gamma+1}\,\frac{1}{M_\infty^2\sin^2\alpha} \tag{2.14}$$

$$\rho = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{\gamma+1}\frac{1}{M_\infty^2\sin^2\alpha}\,\frac{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial\varphi}\right)^2 + \delta^2\left(\frac{\partial R}{\partial x}\right)^2}{\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\frac{1}{R}\frac{\partial R}{\partial\varphi}\right)^2}\right]^{-1} \tag{2.15}$$

3. Introduction of approximate relations. The differential equations and boundary conditions obtained in the preceding section represent an exact formulation of the flow problem. To simplify these relations one can take advantage of the smallness of the parameter characterizing the relative thickness of the body. In conformity with the introduction of dimensionless independent variables in the formulation of the problem, the dependent variables and their derivatives may clearly be regarded as quantities of order unity. Consideration of the first equation of the system (2.3) together with the boundary conditions (2.12) on the shock-wave surface permits one to conclude that in the entire flow field

$$\cot\alpha\, u = \cot\alpha + o(\delta) \tag{3.1}$$

Then discarding quantities of second order of smallness in the differential equations (2.3) and taking (2.4) into account we can write an approximate system of equations for the independent variables v, w, p, ρ

in the form*

$$\begin{aligned}
&\delta \cot\alpha \frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} + \frac{w}{r}\frac{\partial v}{\partial \varphi} - \frac{w^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \\
&\delta \cot\alpha \frac{\partial w}{\partial x} + v\frac{\partial w}{\partial r} + \frac{w}{r}\frac{\partial w}{\partial \varphi} + \frac{vw}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} \\
&\delta \cot\alpha \frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial r} + \frac{w}{r}\frac{\partial \rho}{\partial \varphi} + \rho\left(\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial \varphi} + \frac{v}{r}\right) = 0 \\
&\delta \cot\alpha \frac{\partial}{\partial x}\left(\frac{p}{\rho^\gamma}\right) + v\frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) + \frac{w}{r}\frac{\partial}{\partial \varphi}\left(\frac{p}{\rho^\gamma}\right) = 0
\end{aligned} \tag{3.2}$$

The boundary condition (2.6) on the surface of the body $r = r_1(x, \phi)$ takes the form

$$v - \frac{w}{r_1}\frac{\partial r_1}{\partial \varphi} = \delta \cot\alpha \frac{\partial r_1}{\partial x} \tag{3.3}$$

The boundary conditions on the shock wave surface $r = R(x, \phi)$ become after simplification

$$\begin{aligned}
&(v - \cos\varphi)\frac{1}{R}\frac{\partial R}{\partial \varphi} + w + \sin\varphi = 0 \\
&-\delta\cot\alpha\frac{\partial R}{\partial x} + v - \frac{w}{R}\frac{\partial R}{\partial \varphi} = \frac{\gamma-1}{\gamma+1}\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \frac{\sin\varphi}{R}\frac{\partial R}{\partial \varphi}\right) + \\
&\qquad + \frac{2}{\gamma+1}\frac{1}{M_\infty^2 \sin^2\alpha}\frac{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial \varphi}\right)^2}{-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \frac{\sin\varphi}{R}\frac{\partial R}{\partial \varphi}} \\
&p = \frac{2}{\gamma+1}\frac{\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \frac{\sin\varphi}{R}\frac{\partial R}{\partial \varphi}\right)^2}{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial \varphi}\right)^2} - \frac{1}{\gamma}\frac{\gamma-1}{\gamma+1}\frac{1}{M_\infty^2\sin^2\alpha} \\
&\rho = \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{\gamma+1}\frac{1}{M_\infty^2\sin^2\alpha}\frac{1 + \frac{1}{R^2}\left(\frac{\partial R}{\partial \varphi}\right)^2}{\left(-\delta\cot\alpha\frac{\partial R}{\partial x} + \cos\varphi + \frac{\sin\varphi}{R}\frac{\partial R}{\partial \varphi}\right)^2}\right]^{-1}
\end{aligned} \tag{3.4}$$

Integration of (3.2) with boundary conditions (3.3) and (3.4) gives an approximate solution of the problem posed above of the flow past a slender body at arbitrary angle of attack. We note that, since in all these relations the ratio of the neglected terms to those retained is of order δ^2, the approximation under consideration should provide highly

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* The velocity component u, if required, can be determined by use of Bernoulli's equation.

accurate results (similar to what occurs in hypersonic small-disturbance theory).

4. Flow similitude. If the independent variable x is replaced by the time variable

$$t = \frac{xl}{U_\infty \cos\alpha} \tag{4.1}$$

the approximate relations of the preceding section transform into the differential equations and boundary conditions determining unsteady gas flow in the plane x = const. It is easily seen that this unsteady motion depends on the action of a translating and expanding cylindrical piston. Here, the shape of the piston is determined by the cross-sectional shape of the body, its rate of expansion by the longitudinal distribution of cross-sectional body area, and the velocity of the motion perpendicular to the axis by the angle of attack. This analogy represents a generalization of the equivalence rule ("law of plane sections"), according to which the disturbances produced by a slender body moving with hypersonic speed at arbitrary angle of attack are essentially reduced to the displacement of gas particles in planes perpendicular to the axis of the body.

We now observe that relations (3.2)-(3.4) involve only the two parameters

$$k_1 = \delta\cot\alpha, \qquad k_2 = M_\infty \sin\alpha \tag{4.2}$$

This demonstrates the validity of the similarity rule according to which flows past bodies with similar distributions of area and cross-sectional shape (affinely related bodies) are similar, that is, all the dimensionless functions (v, w, p, ρ) are equal at corresponding points of the field (x, r, ϕ) if the similarity parameters k_1 and k_2 have the same values for two cases.

Using the resulting similarity rule we can, collecting the results, write a formula for the pressure coefficient on the surface of the body in the form

$$C_p = 2\sin^2\alpha \left\{p\left[x,\ r_1(x,\ \varphi),\ \varphi,\ k_1,\ k_2\right] - \frac{1}{\gamma k_2^2}\right\} \tag{4.3}$$

By integrating over the surface of the body it is easy to find expressions: for the normal force coefficient

$$C_n = \frac{N}{{}^1/_2\rho_\infty U_\infty^2 ld} = \sin^2\alpha\, C_n^*(k_1,\ k_2) \tag{4.4}$$

for the axial force coefficient

$$C_t = \frac{\bar{r}_1}{{}^1/_2\,\rho_\infty U_\infty{}^2 ld} = \delta \sin^2\alpha C_t^*(k_1, k_2) \tag{4.5}$$

for the longitudinal moment coefficient

$$C_m = \frac{M}{{}^1/_2\rho_\infty U_\infty{}^2 l^2 d} = \sin^2\alpha C_m^*(k_1, k_2) \tag{4.6}$$

Here, the quantities C_n^*, C_t^*, C_m^*, regarded as functions of the similarity parameters, are equal for affinely similar bodies.

It is easy to see that the equations and boundary conditions of the previous section, as well as the results just formulated, agree, for small angles of attack ($\alpha \sim \delta$), with the well-known relations and results of hypersonic small-disturbance theory for three-dimensional flows [6]. In this sense they can be regarded as a generalization of that theory to arbitrary angle of attack.

5. Aerodynamic characteristics of slender bodies at high angles of attack. At high angles of attack ($\alpha \gg \delta$) one should neglect in the boundary conditions (3.4) not only terms of order δ^2 but also those containing factors of $1/M_\infty^2 \sin^2\alpha$ which, in view of the initial assumption (1.2), have the same or even a higher order of smallness. In this case the solution does not, in general, depend on the Mach number M_∞. This shows that the aerodynamic characteristics of slender bodies at high angles of attack attain far sooner than for small α their hypersonic limits, corresponding to $M_\infty \to \infty$.

The single remaining similarity parameter k_1 becomes small for $\alpha \gg \delta$. This circumstance may be exploited for approximate integration of the system (3.4).

We take advantage of this possibility in order to determine the aerodynamic characteristics of slender bodies whose cross-sectional shape is constant along their length.

The equation of the surface of any such body can clearly be written in the form

$$r = r_1(x, \varphi) = f(x)\,g(\varphi) \tag{5.1}$$

Then the boundary condition (3.3) takes the form

$$v - w\frac{g'(\varphi)}{g(\varphi)} = k_1 f'(x)\,g(\varphi) \tag{5.2}$$

and the boundary conditions (3.4) on the shock-wave surface $r = R(x, \phi)$ can be written in the form

$$v = \cos\varphi - \frac{2}{\gamma+1}\,\frac{-k_1\dfrac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\,\dfrac{1}{R}\,\dfrac{\partial R}{\partial \varphi}}{1 + \dfrac{1}{R}\left(\dfrac{\partial R}{\partial \varphi}\right)^2}$$

$$w = -\sin\varphi + (\cos\varphi - v)\,\frac{1}{R}\,\frac{\partial R}{\partial \varphi} \tag{5.3}$$

$$p = \frac{2}{\gamma+1}\,\frac{\left(-k_1\dfrac{\partial R}{\partial x} + \cos\varphi + \sin\varphi\,\dfrac{1}{R}\,\dfrac{\partial R}{\partial \varphi}\right)^2}{1 + \dfrac{1}{R^2}\left(\dfrac{\partial R}{\partial \varphi}\right)^2}, \qquad \rho = \frac{\gamma+1}{\gamma-1} \tag{5.4}$$

Using the smallness of the parameter k_1 we can, with the degree of approximation adopted, represent the solution of the system (3.2) in the form

$$v = v_0 + k_1 v_1, \quad w = w_0 + k_1 w_1, \quad p = p_0 + k_1 p_1, \quad \rho = \rho_0 + k_1 \rho_1 \tag{5.5}$$

We also represent the equation of the shock-wave surface in the form

$$R(x,\ \varphi) = R_0(x,\ \varphi) + k_1 R_1(x,\ \varphi) \tag{5.6}$$

If we now substitute (5.5) and (5.6) into the differential equations (3.2) and the boundary conditions (5.2), (5.3) and (5.4), it is easy to obtain for the leading terms of (5.5) a system of relations equivalent to the exact formulation of the problem of transverse flow past a cylinder with the Mach number $M_\infty \to \infty$.

Thus

$$R_0(x,\ \varphi) = f(x)\,\sigma_0(\varphi) \tag{5.7}$$

$$v_0 = v_0(y,\ \varphi),\ w_0 = w_0(y,\ \varphi),\ p_0 = p_0(y,\ \varphi),\ \rho_0 = \rho_0(y,\ \varphi) \quad \left(y = \frac{r}{f(x)}\right) \tag{5.8}$$

The linear system of differential equations obtained for the secondary terms in (5.5), together with the corresponding boundary conditions can, as is easily verified, be satisfied by a solution of the form

$$R_1(x,\ \varphi) = f(x)\,f'(x)\,\sigma_1(\varphi) \tag{5.9}$$

$$\begin{aligned} v_1 &= f'(x)\,v_1(y,\ \varphi), \qquad & p_1 &= f'(x)\,p_1(y,\ \varphi) \\ w_1 &= f'(x)\,w_1(y,\ \varphi), \qquad & \rho_1 &= f'(x)\,\rho_1(y,\ \varphi) \end{aligned} \tag{5.10}$$

Collecting results, we obtain the approximate expression

$$C_p = 2\sin^2\alpha\,\{p_0[g(\varphi),\ \varphi] + \delta\cot\alpha f'(x)\,p_1[g(\varphi),\ \varphi]\} \tag{5.11}$$

Then, for the coefficients of aerodynamic forces and longitudinal moment defined above, we find

$$C_n = 2\sin^2\alpha\left[A\int_0^1 f(x)\,dx + B\delta\cot\alpha f^2(1)\right]$$
$$C_t = 2\delta\sin^2\alpha\left[Cf^2(1) + D\delta\cot\alpha\int_0^1 f(x)f'^2(x)\,dx\right] \quad (5.12)$$
$$C_m = 2\sin^2\alpha\left[A\int_0^1 f(x)\,x\,dx + 2B\delta\cot\alpha\int_0^1 f(x)f'(x)\,x\,dx\right]$$

in which the constants A, B, C and D depend only on the cross-sectional shape of the body. Thus, for example, in order to calculate the aerodynamic characteristics of slender bodies of revolution of arbitrary shape it is sufficient to determine these four constants once for all. This can be done either by exact (numerical) integration of the equations obtained above, or by using the results of experimental investigations of a single slender body of revolution (of arbitrary shape) at high supersonic speeds and large angles of attack. The region of applicability of the formulas (5.12) is bounded by the range of angles of attack $\delta \ll \alpha < 1/2\,\pi$.

6. Consideration of real gas properties. Hypersonic flow past a body, particularly at high angle of attack, is associated with the formation of strong shock waves. In the transition across such shock-wave fronts, excitation of additional degrees of freedom of the molecules may take place, in the processes of dissociation and ionization of the gas. Consideration of these effects under the assumption of local thermodynamic equilibrium in the whole flow field presents no difficulties in principle. We will consider the specific entropy $S°$ and specific enthalpy $h°$ of the gas to be functions of the pressure and density, and also the thermodynamic state and chemical composition of the gas in the undisturbed stream to be characterized by the values p_∞, ρ_∞, and the concentrations $C_{i\infty}$ of the components [7].

Then, together with relations (2.4) and (2.10), we have the relations

$$S° = S_\infty S(p°, \rho°;\ p_\infty, \rho_\infty, C_{i\infty}) = S_\infty S\,(p\rho_\infty a_\infty^2 M_\infty^2 \sin^2\alpha,\ \rho\rho_\infty;\ p_\infty, \rho_\infty, C_{i\infty}) \quad (6.1)$$
$$h° = h_\infty h\,(p°, \rho°;\ p_\infty, \rho_\infty, C_{i\infty}) = h_\infty h\,(p\rho_\infty a_\infty^2 M_\infty^2 \sin^2\alpha,\ \rho\rho_\infty;\ p_\infty, \rho_\infty, C_{i\infty}) \quad (6.2)$$

which we can clearly put into the simpler forms

$$S° = S_\infty S\,(k_2^2 p, \rho;\ p_\infty, \rho_\infty, C_{i\infty}) \quad (6.3)$$
$$h° = h_\infty h\,(k_2^2 p, \rho;\ p_\infty, \rho_\infty, C_{i\infty}) \quad (6.4)$$

From this follows the possibility of generalizing the results obtained above for the similitude of hypersonic flows of a perfect gas with constant specific heats to the general case of flows in thermodynamic equilibrium. For similitude of flows past a family of affinely-related bodies in this case, aside from the constancy of the similarity parameters k_1 and k_2, the conditions must also be fulfilled of the same chemical composition and thermodynamic state in the undisturbed stream.

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